Measurements of the return loss at the input and output ports indicated a slight mismatch at the design frequencies, partially due to extrapolating the transistor model from 18 to 30 GHz. However, processing factors that were unknown when the circuit was designed resulted in a suboptimal Lange coupler, and an inferior match at the RF and LO ports.

The return loss is shown in Figs. 4 and 5. As can be seen, at the frequencies of minimum conversion loss, the return loss was approximately -15, -10, and -15 dB at the LO, RF, and IF ports, respectively.

The third-order intercept was measured at an output power of approximately -6.2 dBm, corresponding to an input power of 4.5 dBm, under the conditions that provided maximum conversion gain. Fig. 6 shows the third-order intercept. Note that closely spaced measurements were made at low power levels due to power limitations of the sources used.

The LO–RF isolation was measured to be 49 dB, the RF–IF isolation was 52 dB, and the LO–IF isolation was 33 dB at the frequency of minimum conversion loss.

This mixer's performance is similar to the published performance of other mixers; this is significant because the 0.8-μm MESFET process used for this mixer is much less expensive than most of the processes usually used at this frequency. Table I shows a comparison of this work with other reported mixers. Note that the data for this work are measured using only a single output.

IV. Conclusions

This paper has presented a MESFET downconvert mixer that uses the common-gate configuration. The common-gate configuration allows a simpler process to be used than would be possible with a CS configuration. Measurements on the mixer indicate a conversion loss of 10.7 dB, with a third-order intercept at -6.2-dBm output power.

A survey of published mixer results showed that the performance of this mixer is similar to that of other mixers in this frequency band. However, most mixers designed in the Ka-band use more expensive higher performance processes. This paper suggests that circuits may be designed using common-gate transistors to achieve performance comparable to circuits designed using more expensive processes.

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References

[1] P. Bura and R. Dikshit, "F.E.T. mixer with the drain L.O. injection," *Electron. Lett.*, vol. 12, pp. 536–537, Sept. 1976.

[2] A. Minakawa and T. Hirota, "An extremely small 26 GHz monolithic image-rejection mixer without DC power consumption," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1634–1637, Sept. 1993.

[3] R. Pucel, D. Mass, and R. Bera, "Performance of GaAs MESFET mixers at X-band," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 351–360, June 1976.

[4] C. Peñalosa and C. Aitchison, "Analysis and design of MESFET gate mixers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 643–652, July 1987.

[5] M. Madihian, L. Desclos, K. Maruhashi, K. Onda, and M. Kuzuhara, "A K-band monolithic CPW upconverter utilizing a source mixing concept," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1995, pp. 127–130.

[6] P. Shearing, "A coplanar Ka-band balanced common gate amplifier in GaAs MESFET," M.S. thesis, Dept. Elect. Comput. Eng., Royal Military College Canada, Kingston, ON, Canada, 1997.

[7] P. Shearing, A. Freundorfer, and Y. Antar, "A coplanar Ka-band common-gate amplifier in 0.8 mm GaAs MESFET," in *Antenna Technol. Appl. Electromagn. Symp.*, Ottawa, ON, Canada, 1998, pp. 61–64.

[8] M. L. de la Fuente, J. Portilla, J. Pascual, and E. Artal, "Low-noise Ku-band MMIC balanced P-HEMT upconverter," *IEEE J. Solid-State Circuits*, vol. 34, pp. 259–263, Feb. 1999.

[9] B. Frank, "A novel common gate mixer for wireless applications," Dept. Elect. Comput. Eng., M.S. thesis, Queen's Univ., Kingston, ON, Canada, 1999.

[10] F. Cardinal, H. An, I. Mag, and R. Smith, "A high-performance broadband MMIC PHEMT resistive drain mixer for 28–40 GHz band PCN applications," in *IEEE Microwave Millimeter-Wave Monolithic Circuits Symp. Dig.*, 1996, pp. 47–50.

[11] K. Chang, E. Lin, H. Wang, K. Tan, and W. Ku, "A W-band monolithic, singly balanced resistive mixer with low conversion loss," *IEEE Microwave Guided Wave Lett.*, vol. 4, pp. 301–305, Sept. 1994.

[12] T. Chen, T. Ton, G. Dow, K. Nakano, L. Liu, and J. Berenz, "A Q-band monolithic balanced resistive HEMT mixer using CPW/slot-line balun," in *IEEE GaAs IC Symp. Dig.*, 1990, pp. 177–180.

[13] A. Freundorfer and C. Falt, "A Ka-band GaInP/GaAs HBT double balanced upconvert mixer using lumped element balun," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1996, pp. 578–581.

[14] K. Kobayashi, A. Oki, D. Umemoto, T. Block, and D. Streit, "A novel monolithic HEMT-HBT Ka-band VCO-mixer design," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1997, pp. 83–86.

[15] S. Nam, T. Gokdemir, A. Baree, I. Robertson, A. Plews, M. Howes, C. Snowden, J. Leckey, A. Patterson, and J. Stewart, "Fully integrated Q-band MMIC transmitter and receiver chips using resistive PHEMT mixers," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1998, pp. 279–282.

[16] M. Schefer, U. Lott, H. Benedickter, H. Meier, W. Patrick, and W. Bächtold, "Active, monolithically integrated coplanar V-band mixer," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1997, pp. 1043–1046.

[17] J. Wenger and J. Splettstößer, "Ka- and V-band MMIC components for personal communication networks," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1996, pp. 491–494.

[18] K. Yhland, N. Rorsman, and H. Zirath, "Novel single device balanced resistive HEMT mixers," in *IEEE Trans. Microwave Theory Tech.*, vol. 43, Dec. 1995, pp. 2863–2867.

Analytical Evaluation of the MoM Matrix Elements for the Capacitance of a Charged Plate

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***Abstract*—Closed-form expression is derived for the fourfold integral involved in the evaluation of the capacitance of a charged plate using the Galerkin's procedure in the method of moments. The dimensions of each rectangular subsection for discretizing the conducting plate can be arbitrary. The calculated solutions converge faster than the point-matching results, as expected.**

***Index Terms*—Galerkin's procedure, method of moments.**

I. Introduction

The capacitance of a charged conducting plate can be evaluated by the method of moments (MoM). Thus far, in the MoM, the

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$$\int_{x_m-(\Delta_m/2)}^{x_m+(\Delta_m/2)} \int_{y_m-(\delta_m/2)}^{y_m+(\delta_m/2)} \int_{x_n-(\Delta_n/2)}^{x_n+(\Delta_n/2)} \int_{y_n-(\delta_n/2)}^{y_n+(\delta_n/2)} \frac{dy'\,dx'\,dy\,dx}{4\pi\varepsilon_0\sqrt{(x-x')^2+(y-y')^2}} = \sum_{i=1}^{4}\sum_{j=1}^{4}(-1)^{s(i,j)}\frac{f(\alpha_i,\beta_j)}{4\pi\varepsilon_0} \quad (1)$$

point-matching technique has been used [1]. The basis functions are localized two-dimensional pulse functions, and the test functions are two-dimensional Dirac's delta functions. It follows that each MoM matrix element is a two-dimensional integral. In [1], the integral is approximately evaluated as if the charge distribution over a divided subsection was a point charge. Recently, a closed-form integration has been derived in [2] and the final result for the capacitance appears to be reliably convergent.

In the MoM, the main advantage of the point-matching technique is that the evaluation of the matrix elements can be greatly simplified since the Dirac's delta functions are involved in the integration defined for the inner product of the problem. The major disadvantage is that, for low-order solutions, the accuracy and convergence of the solution generally depend on the location of the matching points [3]. In the point-matching analysis of a cylindrical dipole in [4], it is reported that the positions of the matching points have to be carefully selected away from the regions of zero fields produced by the basis functions. The Galerkin's procedure, on the other hand, has been found to give better results and faster convergence in the majority of cases for higher order solutions [3]. This motivates us to derive the closed-form integration for the Galerkin's solution of the charged plate.

II. MoM Matrix Elements

When the Galerkin's procedure in the MoM is employed to calculate the capacitance of a conducting plate in free space, the inner product involved in the method becomes a fourfold integral. The simple subsection method [1] can be used to divide the plate into $M \times N$ rectangular subsections. Let the center of each subsection be (x_k, y_k), $k = 1, 2, \ldots, M \times N$, and the associated basis function be a two-dimensional pulse, which is unity over $x \in [x_k - \Delta_k/2, x_k + \Delta_k/2]$ and $y \in [y_k - \delta_k/2, y_k + \delta_k/2]$. It can be shown that the closed-form result for the fourfold integral is (1), shown at the top of this page, where

$$f(\alpha, \beta) = \frac{\alpha\beta}{4}\left\{\beta \ln \frac{\sqrt{\alpha^2+\beta^2}+\alpha}{\sqrt{\alpha^2+\beta^2}-\alpha} + \alpha \ln \frac{\sqrt{\alpha^2+\beta^2}+\beta}{\sqrt{\alpha^2+\beta^2}-\beta}\right\} - \frac{1}{6}\sqrt{(\alpha^2+\beta^2)^3}$$

$$s(i, j) = \begin{cases} 0, & i = j \text{ or } i + j = 5 \\ 1, & \text{otherwise.} \end{cases}$$

$$\begin{aligned}
\alpha_1 &= X_{mn} + \Delta_m/2 + \Delta_n/2 \\
\beta_1 &= Y_{mn} + \delta_m/2 + \delta_n/2 \\
\alpha_2 &= X_{mn} + \Delta_m/2 - \Delta_n/2 \\
\beta_2 &= Y_{mn} + \delta_m/2 - \delta_n/2 \\
\alpha_3 &= X_{mn} - \Delta_m/2 + \Delta_n/2, \\
\beta_3 &= Y_{mn} - \delta_m/2 + \delta_n/2 \\
\alpha_4 &= X_{mn} - \Delta_m/2 - \Delta_n/2 \\
\beta_4 &= Y_{mn} - \delta_m/2 - \delta_n/2 \\
X_{mn} &= x_m - x_n \\
Y_{mn} &= y_m - y_n.
\end{aligned}$$

In (1), $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is used.

TABLE I
Capacitance (in Picofarads) of a Unit Square Conducting Plate (1 m × 1 m)

L	Point-matching [2]	Galerkin's solution
1	35.1765 (13.8%)	37.4217 (8.30%)
2	37.7355 (7.53%)	38.9394 (4.58%)
3	39.1877 (3.97%)	39.7938 (2.49%)
4	39.9726 (2.05%)	40.2751 (1.31%)
5	40.3828 (1.05%)	40.5337 (0.67%)
6	40.5939 (0.53%)	40.6688 (0.34%)
Extrapolated	40.8097	40.8087

III. Numerical Results

Table I compares the capacitance values for a 1 m $\times$ 1 m conducting plate calculated by (1) with those obtained by the point-matching technique [2]. The two dimensions of the plate are equally partitioned into $M = N = 2^L$ subsections, and the matrix size is $M^2 \times M^2$. In our calculation, the size of the final matrix is reduced to $(M/2)^2 \times (M/2)^2$ by utilizing the structural symmetry.

The last row of Table I shows the Richardson's extrapolation results to $L = 6$. The relative deviation between the two extrapolated values is less than 2.5 $\times$ 10^{-5}. It is plausible to assume that these extrapolated values are the converged results. In Table I, the number in the parentheses is the relative deviation of each calculated value from the converged result for each set of solutions. Obviously, the Galerkin's results have faster convergence than the point-matching solutions for this particular study.

IV. Conclusion

We have presented the closed-form expression for the fourfold integral involved in the MoM calculation of the capacitance of a conducting plate. The expression is applicable to nonuniform rectangular subsections in two dimensions. The calculations show how the Galerkin's procedure provides faster convergence than the point-matching solution to this particular problem.

References

[1] R. F. Harrington, *Field Computation by Moment Methods*. Melbourne, FL: Krieger, 1968.

[2] R. Bancroft, "A note on the moment method solution for the capacitance of a conducting flat plate," *IEEE Trans. Antennas Propagat.*, vol. 45, p. 1704, Nov. 1997.

[3] M. M. Ney, "Method of moments as applied to electromagnetic problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 972–980, Oct. 1985.

[4] T. K. Sarkar, A. R. Djordjevic, and E. Arvas, "On the choice of expansion and weighting functions in the numerical solution of operator equations," *IEEE Trans. Antennas Propagat.*, vol. AP-33, pp. 988–996, Sept. 1985.